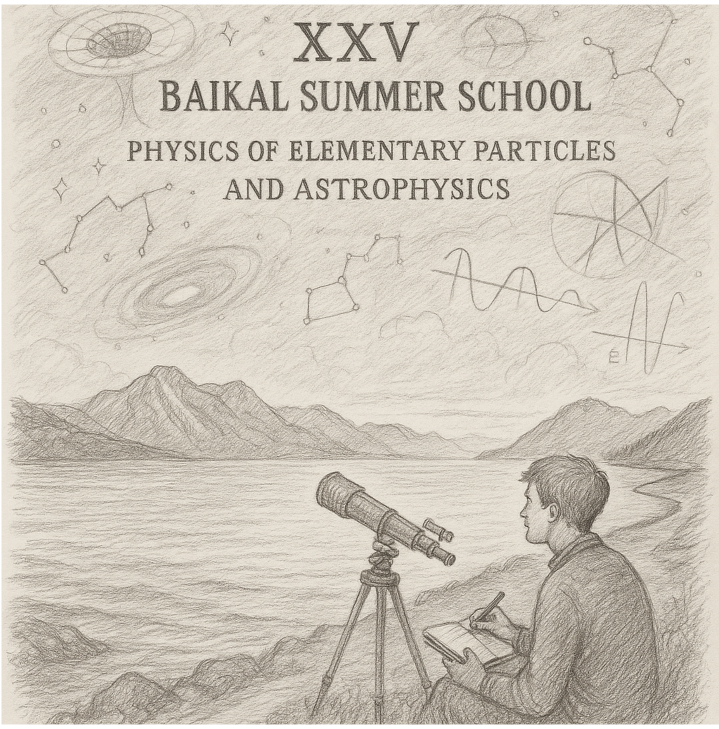
# Student's Projects
# XXV Baikal Summer School
# on Physics of Elementary Particles and Astrophysics

July, 11-18 2025



# Contents

# Project 1

# Spacetime Geometry and Observer Kinematics

*Author: Prof. Emil Akhmedov*

1. Derive the metric as observed by a freely falling observer in the Schwarzschild spacetime. Provide a physical interpretation of the resulting metric. What does it reveal about the observer's experience of spacetime near and inside the event horizon?

2. Sketch the worldline of an eternally uniformly accelerating observer on the Penrose diagram of flat (Minkowski) spacetime. How does the diagram change if the acceleration occurs only for a finite duration? Identify and describe the regions of spacetime that are causally accessible to the observer in each case.

3. Explore a real-world analog gravity experiment that simulates features of curved spacetime or event horizons:

   - **BEC Sonic Horizon:** Review the experiment by Steinhauer [1], which realizes a sonic black hole in a Bose-Einstein condensate. Describe how the experiment models Hawking radiation and how it relates to observer-dependent horizons.
   - **Optical Event Horizon:** Investigate the experiment by Belgiorno et al. [2], which demonstrates photon emission from an optical horizon in a nonlinear fiber. Discuss how soliton pulses mimic gravitational horizons.
   - **Water Tank Analogy:** Study water wave analogs of black holes as presented by Weinfurtner et al. [3]. Analyze how surface wave blocking models a horizon and how this affects signal propagation.

- Optional: Reproduce part of the simulation or propose a simplified classroom demonstration based on one of these setups.

## Deliverables

- Calculations and plots: Derivations of coordinate transformations, Penrose diagram sketches, and causal structure illustrations.
- Scientific Report: A concise LaTeX-written report that includes full derivations, physical interpretations, and visualizations of the spacetime structure for freely falling and accelerating observers.
- Presentation: A 10-minute talk summarizing the observer-dependent geometry of Schwarzschild and Minkowski spacetimes.

## References

1. *Steinhauer J.* Observation of quantum Hawking radiation and its entanglement in an analogue black hole // Nature Physics. — 2016. — Vol. 12. — P. 959–965. — DOI: `10.1038/nphys3863`.
2. Hawking radiation from ultrashort laser pulse filaments / F. Belgiorno [et al.] // Physical Review Letters. — 2010. — Vol. 105, no. 20. — P. 203901. — DOI: `10.1103/PhysRevLett.105.203901`.
3. Measurement of stimulated Hawking emission in an analogue system / S. Weinfurtner [et al.] // Physical Review Letters. — 2011. — Vol. 106, no. 2. — P. 021302. — DOI: `10.1103/PhysRevLett.106.021302`.

# Project 2

# Dark Matter in the Galaxy and Direct Detection

*Author: Prof. Jianglai Liu*

1. The measured rotational curve of our galaxy is shown in Fig. **??**. Assuming that the dark matter halo dominates the entire galaxy and its distribution has spherical symmetry, determine the mass distribution function $M(r)$ of the dark matter halo as a function of the distance $r$ to the galactic center. What is the local dark matter density (mass per unit volume) at the location of the Sun?

2. Based on the same rotation curve, assume the dark matter particle has a mass of $100\,\mathrm{GeV}/c^2$. Estimate the typical energy deposited when such a dark matter particle elastically scatters off:

   - a xenon nucleus,
   - an argon nucleus,
   - and an electron.

3. In liquid xenon detectors, nuclear recoil energy is subject to "quenching" and requires careful calibration. Design a neutron calibration experiment that produces xenon-nucleus recoil energies similar to those from dark matter scattering. Estimate the typical kinetic energy of the neutrons required.

4. Assume a spin-independent elastic scattering cross section of $\sigma = 10^{-42}\,\mathrm{cm}^2$ and a dark matter mass of $100\,\mathrm{GeV}/c^2$. How many dark matter-nucleus scattering events are expected in a liquid xenon experiment with an exposure of $1\,\mathrm{ton}\cdot\mathrm{year}$, assuming a local dark matter density of $\rho_\chi = 0.3\,\mathrm{GeV}/\mathrm{cm}^3$ and an average velocity $v \sim 220\,\mathrm{km/s}$?

5. Suppose a background-free experiment with a 1 ton · year exposure observes zero candidate events. What is the 90% confidence level upper limit on the dark matter–xenon nucleus scattering cross section?

6. Alternatively, consider an experiment where 9 events are observed with 9 expected background events. What is the 90% confidence level upper limit on the signal rate? What is the corresponding upper limit on the dark matter–xenon cross section?

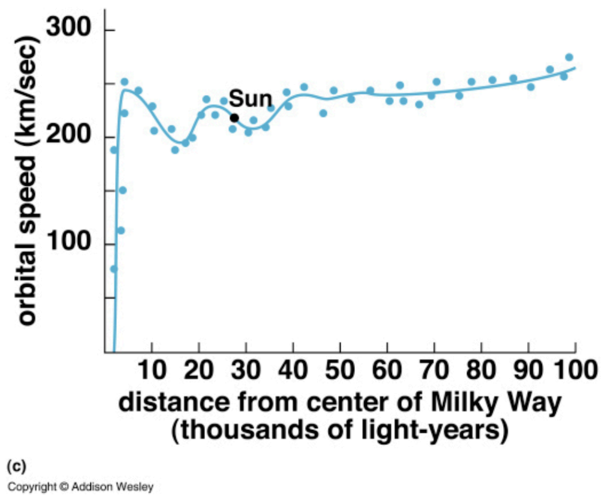



Figure 2.1: Observed rotational curve of the Milky Way Galaxy

# Deliverables

- Calculations and plots: Rotation curve analysis, derived halo mass distribution, recoil energy spectra for different targets, neutron calibration energy estimates, and event rate predictions.
- Scientific Report: A concise LaTeX-written report detailing the theoretical derivations, numerical estimates, and implications for dark matter detection.
- Presentation: A 10-minute talk summarizing the modeling of the galactic halo, dark matter detection principles, and experimental constraints.

# Project 3

# Annihilating Dark Matter

*Author: Sergey Zavyalov*

Dark matter (DM) constitutes one of the posible extensions of the Standard Model (SM). Existing SM particles cannot account for the required DM densities and masses. However, robust astronomical evidence supports DM existence, including galactic rotation curves and gravitational lensing in colliding galaxy clusters (e.g., the Bullet Cluster). These observations necessitate physics beyond the SM.

DM is classified as cold (CDM) or hot (HDM). CDM provides the best fit to observed large-scale structure and CMB data, with Weakly Interacting Massive Particles (WIMPs) and axions as leading candidates. Neutrinos can represent HDM but contribute negligibly to the total DM density due to insufficient mass and abundance; they additionally suppress small-scale structure formation.

This project proposes a multi-messenger astronomy approach to probe DM signatures. We will investigate:

i. Sterile neutrino decay into active neutrinos and photons

ii. CDM annihilation to photons and neutrinos

iii. Combined analysis of both channels

**Tasks:**

**i. Sterile Neutrinos: Theory**

- Examine the seesaw mechanism for neutrino mass generation. Construct the most general one flavour Yukawa Lagrangian including Dirac and Majorana mass terms. Demonstrate that the low-energy effective action reduces to a single dimension-5 operator (Weinberg operator). Predict neutrino masses assuming a GUT scale ($\sim 10^{15}$ GeV).

- Analyze radiative neutrino decay (Dirac/Majorana) $\nu_2 \to \nu_1 + \gamma$. Depict Feynman diagrams and decay widths for both cases. Provide numerical estimates.

**ii. Sterile Neutrinos: Experiment**

- Compile galactic x-ray data in the energy range relevant for sterile neutrino decay. Model Galactic DM density profiles and subtract astrophysical foregrounds.
- Isolate background and signal regions using established signal/background techniques. Extract signal photon spectra from sterile neutrino decays.
- Simulate a space telescope with characteristic energy resolution, efficiency, exposure, and field of view (based on existing observatories).
- Constrain model parameters (neutrino mixing angle, sterile neutrino mass) via likelihood maximization or equivalent methods. Compare results with existing experimental limits.

**iii. Annihilating Cold Dark Matter: Photons**

- Compute expected photon spectra from CDM annihilation in the Galactic halo.
- Repeat analysis from paragraph (ii.) to constrain DM annihilation cross-section and mass.

**iv. Annihilating Cold Dark Matter: Neutrinos**

- Compute expected neutrino spectra from CDM annihilation in the Sun's gravitational potential.
- Repeat analysis from paragraph (ii.) using a simple neutrino telescope model (based on IceCube, Baikal-GVD, or KM3NeT). Constrain annihilation cross-section and mass of CDM particles.
- Perform combined sensitivity analysis using photon and neutrino channels.

**Deliverables:**

1. *Scientific Report*: Concise LaTeX report detailing methodology, results, uncertainties, and implications

2. *Presentation*: 10–15 minute summary of approach, constraints, and astrophysical implications

# Project 4

# The detectability of other neutrino sources like NGC 1068

*Author: Prof. Lili Yang*

## Introduction

The IceCube experiment reported an excess neutrinos associated with the nearby active galaxy NGC 1068 with a significance of 4.2 $\sigma$. NGC 1068 is the brightest and one of the closest Serfert galaxies, and has been seen by various instruments. With the analysis of multi-messenger observations, the mechanisms and production regions of neutrinos can be understood.

## Tasks

1. **Build a numerical model**

   - Obtain the spectral energy distribution (SED) of NGC 1068, including electromagnetic radiation from radio to gamma-ray bands and neutrinos.

   - Model the SED with reasonable setup and present the parameters involved.

2. **Calculate neutrino flux**

   - Search for potential neutrino candidates of Serfert galaxies located in the Southern Hemisphere, and make a comparison of their physical properties.

- Apply the constructed model to these selected sources, and calculate their neutrino fluxes arriving at Earth.

3. **Estimate the event rates**

- Calculate the $\nu_\mu + \bar{\nu}_\mu$ event rates from these sources and atmospheric background for KM3NeT.

- To identify the sources, getting a significance of 5 $\sigma$, assess the time needed.

4. **Impact of the factors involved**

- Analyze how the effective area and angular resolution affect the detectability of the source.

- Analyze how the distance and luminosity of sources affect their detectability.

# Deliverables

- Calculations and plots: SED and fitted model, predicted neutrino flux, event rates, detectability of neutrino sources with various distance and luminosity.

- Scientific Report: a concise LaTeX-written report that presents the source modeling, interpretation, results, and impact of the detector parameters and sources properties.

- Presentation: a 10-minute talk summarizing the constructed model and neutrino detectability.

# Project 5

# High-Energy Astrophysical Neutrinos

*Author: Vladimir Allakhverdian*

1. Assume the photon spectrum is given by:

$$\frac{dN}{dE} = A \cdot \frac{E^{-\gamma}}{1 + \tan\left(\frac{\pi E}{2E_{\mathrm{cut}}}\right)}, \tag{5.1}$$

   Determine, in the simplest approximation, the corresponding spectrum of muon neutrinos $\nu_\mu$, assuming the neutrinos originate from the reaction chain:

$$p + p \longrightarrow \pi^{\pm} + \ldots \longrightarrow \nu_\mu, \bar{\nu}_\mu, \ldots$$

   *Tip: estimate the average fraction of energy transferred from protons to neutrinos in these processes. It may be useful to treat the combined $\nu_\mu + \bar{\nu}_\mu$ flux rather than each separately. See e.g., arXiv:2212.11236 for reference.*

2. Select a specific astrophysical source on the sky and determine the normalization constant $A$ using the luminosity of this source.

   *If a specific source is difficult to work with, make a rough estimate by selecting a representative energy (e.g., 100 TeV) and comparing your model flux to IceCube measurements at that energy.*

3. Compare your estimate of the diffuse neutrino flux with the experimental measurements from IceCube.

   *Optional: Plot the model proton and neutrino fluxes together with IceCube data points.*

4. Treat $\gamma$ and $E_{\text{cut}}$ as free parameters of the model. Using available data, determine the best-fit values.

   *Use a chi-square goodness-of-fit test and display two-dimensional confidence contours for the parameters* $(\gamma, E_{\text{cut}})$ *corresponding to* $1\sigma$*,* $2\sigma$*, etc.*

# Deliverables

- Calculations and plots: Derived neutrino spectrum, comparison with IceCube data, and overlaid fluxes for protons and neutrinos. If applicable, include best-fit parameter contours for $\gamma$ and $E_{\text{cut}}$.
- Scientific Report: A concise LaTeX-written report with all derivations, model assumptions, flux estimates, data comparison, and parameter fit results.
- Presentation: A 10-minute talk summarizing the model setup, astrophysical motivation, numerical results, and comparison with observational data.

# Project 6

# Stellar Evolution

*Author: Vladimir Allakhverdian*

1. Write down the simplified system of equations governing stellar structure:

    - mechanical equilibrium,
    - equation of state,
    - energy transport,
    - energy generation,
    - mass conservation.

    *Tip: choose a convenient radial variable — either radius or enclosed mass. Derive the mass conservation law from the definition of a differential mass shell. The equation of state may be taken as an ideal gas plus radiation pressure. Mechanical equilibrium balances gravity and pressure gradients via Newton's second law. The remaining two equations follow from definitions of luminosity and specific energy generation per unit mass.*

2. Specify appropriate boundary conditions at the center and surface of the star. At both boundaries, mass, radius, and luminosity are well-defined. At the surface, you may also impose a given stellar temperature.

3. Numerically solve the system and estimate the lifetime of the star as a function of its mass and luminosity.

    *Optional: Plot stellar lifetime as a function of mass and luminosity on a 2D diagram. Estimate the main sequence lifetime of the Sun from this plot.*

# Deliverables

- Calculations and plots: Complete derivation of the stellar structure equations, boundary condition formulation, and numerical solutions for stellar lifetime. Include plots showing lifetime as a function of mass and luminosity, and mark the Sun on the diagram.
- Scientific Report: A concise LaTeX-written report presenting the assumptions, model equations, numerical scheme, and physical interpretation of results.
- Presentation: A 10-minute talk summarizing the equations, boundary conditions, numerical solutions, and main physical insights about stellar evolution.

# Project 7

# Possible location of neutrinos in blazars basing on core shift measurements

*Author: Prof. Elena Nokhrina*

## Tasks:

1. **Sample construction**: Compile a sample of bright blazars — possible neutrino sources — with measured core shift, viewing angle and apparent speed.
2. **Estimates of flare delays between 11 and 22 GHz cores**:
   - Basing on two models for a core shift index, calculate an expected time delay between 11 and 22 GHz cores.
   - Compare with the observed median delay for suspected neutrino sources.
3. **Discuss possible reasons of the descrepancies**:
   - Neutrino source upstream the 22 GHz core.
   - Flare speed is different from bright components kinemateics speed estimate.
   - Analyze the possible uncertainty in viewing angle estimate.
4. **Physical conditions in a region of neutrino origin**: Estimate, where it is possible, the emitting plasma number density and magnetic field in a potential region of neutrino origin.

# Derivables:

- **Results**: a well organized table with sources properties and derived values.
- **Scientific report**: a concise LaTeX-written report summarizing sample constrction, assumptions, results and conclusions.
- **Presentation**: short presentation on methods and results.

# Project 8

# Neutrino Oscillation in Matter

*Author: Vladimir Allakhverdian*

1. Consider a two-flavor approximation for neutrino oscillations. Write down the evolution equation for the neutrino state vector in matter.

   *Hint: Express the neutrino as a two-component vector in the flavor basis. Write the total Hamiltonian including both vacuum oscillations and the effective potential due to interactions with matter. Remember that you have two natural bases: the mass basis (diagonal in energy) and the flavor basis (diagonal in interaction Hamiltonian).*

2. Specify a model for the solar density profile.

   *Options:*

   - Fit a simple exponential form $\rho(r) = A \cdot \exp(-r/r_0)$ to published solar data.
   - Use a best-fit model (e.g. NASA's Standard Solar Model or tabulated profiles from Bahcall 1989).

   *Plot several density models and discuss their differences. You'll use them in your numerical simulation.*

3. Assuming electron neutrinos are produced in the solar core, determine their quantum state upon reaching the solar surface.

   *Compute or numerically integrate the evolution matrix. Apply it to an initial pure $\nu_e$ state at the solar center.*

4. Estimate the survival probability of electron neutrinos as a function of their energy.

*Use values of $\delta m^2_{12}$ and mixing angle from the Particle Data Group. Compare survival probabilities for different solar density models. Plot $P_{ee}(E)$.*

## Deliverables

- Calculations and plots: Hamiltonian in matter, density profile models, numerical evolution of the neutrino state, and survival probability as a function of energy. Include comparisons of different solar density models.
- Scientific Report: A concise LaTeX-written report detailing the theory, derivation, modeling assumptions, and numerical results.
- Presentation: A 10-minute talk presenting the oscillation mechanism in matter, model comparisons, and survival probability predictions relevant to solar neutrino experiments.

# Project 9

# Hawking Radiation and Primordial Black Holes

*Author: Prof. Dmitry Naumov*

1. In the Hawking radiation model, a black hole loses energy by emitting photons. Bekenstein and Hawking derived an estimate for the power of this radiation:
$$P = \frac{\hbar c^6}{15360\pi G^2 M^2} = \frac{dMc^2}{dt}. \tag{9.1}$$
Estimate the total evaporation time $t$ of a black hole as a function of its initial mass $M$. Compute $t$ for:

   - $M = 2 \cdot 10^{30}$ kg (the mass of the Sun),
   - $M = 10^{11}$ kg.

   Compare your results with the current age of the Universe ($1.4 \cdot 10^{10}$ years).

2. Suppose a microscopic black hole is produced in a proton-proton collision at the LHC with energy 13 TeV. Estimate its lifetime due to Hawking evaporation. Would it have time to grow by absorbing surrounding matter (e.g., Earth)?

3. From task (1), the evaporation time of a black hole with $M = 10^{11}$ kg is comparable to the age of the Universe. Some astronomers search for such evaporating primordial black holes. However, the Universe is filled with cosmic microwave background radiation at temperature $T = 2.7$ K. For a black hole to evaporate, its Hawking temperature $T_H$ must exceed 2.7 K.

The temperature of a black hole is:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}. \tag{9.2}$$

Estimate the maximum mass a black hole can have and still evaporate in the presence of the cosmic microwave background. What observational consequences might this have?

## Deliverables

- Calculations and plots: Black hole lifetime as a function of mass, Hawking temperature vs. mass, and comparison with the age and temperature of the Universe. Include estimates for specific cases and plots illustrating thresholds for evaporation.
- Scientific Report: A concise LaTeX-written report explaining the Hawking radiation mechanism, derived expressions, evaporation conditions, and implications for primordial black holes as observable phenomena.
- Presentation: A 10-minute talk summarizing the key physics of black hole evaporation, lifetime estimates, cosmological relevance, and potential observational signatures.

# Project 10

# Particle Acceleration at Astrophysical Shocks

*Author: Prof. Evgeny Derishev*

1. Consider a strong shock wave propagating into a medium with a known equation of state. Under the assumption that the upstream pressure is negligible (strong shock approximation), solve the conservation equations for mass, momentum, and energy across the shock front. Determine the velocity of the downstream flow relative to the shock.

2. Develop a numerical code to simulate the trajectory of a test particle undergoing random scattering in the vicinity of the shock. Model the system by dividing space into upstream and downstream regions, each with different bulk velocities. Implement the following assumptions:

   (a) The scattering probability per unit time is a specified function of the particle's position.

   (b) Each scattering is isotropic in the rest frame of the local fluid.

   (c) The simulation ends when the particle travels sufficiently far into the downstream region.

3. Run the simulation multiple times with varied initial conditions for the particle's position and velocity. Accumulate statistics over a large number of simulated particles.

4. Using the statistical data obtained:

   - Calculate the average rate of energy gain per scattering cycle.
   - Compute the energy distribution of particles escaping downstream.

  - Compare your results to expectations from first-order Fermi acceleration theory.

## Deliverables

- Calculations and plots: Shock jump conditions, downstream velocity estimates, particle trajectories, energy gain per cycle, and final energy distributions. Include comparisons with theoretical predictions of Fermi acceleration.
- Scientific Report: A concise LaTeX-written report describing the physical model, derivations, numerical simulation methods, statistical results, and their interpretation in the context of shock acceleration theory.
- Presentation: A 10-minute talk outlining the acceleration mechanism, simulation setup, key results, and comparison to Fermi acceleration expectations.

# Project 11

# How to observe a Wormhole

*Author: Prof. Dmitry Naumov*

1. To understand how a wormhole distorts visual perception, place a virtual camera near its throat and analyze the light rays reaching it. There are exactly two classes of photon trajectories:

   - *Regular geodesics*, which remain in our Universe (the upper half of the wormhole surface),
   - *Singular geodesics*, which pass through the wormhole and come from the "other Universe" (the lower half).

   These trajectories can be found by solving the geodesic equation in General Relativity:
   $$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \tag{11.1}$$
   where $s$ is the affine parameter and $\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols determined by the geometry. They encode how space curves and do not depend on the particle.

2. Due to azimuthal symmetry of the wormhole, derive the conserved angular momentum:
   $$V_\varphi = g_{\varphi\varphi}V^\varphi = r^2\frac{d\varphi}{ds} = L = \text{const.} \tag{11.2}$$

3. Show that the squared norm of the velocity vector gives:
   $$V_iV^i = g_{rr}\left(\frac{dr}{ds}\right)^2 + \frac{L^2}{r^2} = C^2 > 0, \tag{11.3}$$

and derive the first-order differential equation for the radial coordinate:

$$\left(\frac{dr}{ds}\right)^2 = \frac{1}{g_{rr}}\left(C^2 - \frac{L^2}{r^2}\right). \tag{11.4}$$

Define the turning point $r = r_t$ by the condition $\frac{dr}{ds} = 0$, leading to:

$$r_t^2 = \frac{L^2}{C^2}. \tag{11.5}$$

Change variables from $s$ to $\varphi$ and show that:

$$\left(\frac{dr}{d\varphi}\right)^2 = \frac{r^2}{g_{rr}}\left(\frac{r^2}{r_t^2} - 1\right). \tag{11.6}$$

4. For a wormhole with metric coefficient $g_{rr} = \left(1 - \frac{b^2}{r^2}\right)^{-1}$, derive the integral form of the geodesic trajectory:

$$\varphi(r) = \int \frac{dr}{\sqrt{(r^2 - b^2)\left(\frac{r^2}{r_t^2} - 1\right)}}. \tag{11.7}$$

5. Obtain the **regular solution** in terms of Jacobi elliptic functions:

$$r(\varphi) = r_t \cdot dc\left(\varphi, \frac{b^2}{r_t^2}\right), \tag{11.8}$$

where $dc = dn/cn$, and $dn$, $cn$ are Jacobi elliptic functions. The angular coordinate $\varphi \in (-\varphi_r, \varphi_r)$, where $\varphi_r = K(b^2/r_t^2)$, and $K$ is the complete elliptic integral of the first kind.

6. Derive the **singular solution**:

$$r(\varphi) = b \cdot dc\left(\frac{b\varphi}{r_t}, \frac{r_t^2}{b^2}\right), \tag{11.9}$$

with $\varphi \in (-\varphi_s, \varphi_s)$, where $\varphi_s = \frac{r_t}{b} \cdot K(r_t^2/b^2)$.

7. Plot both regular and singular geodesic trajectories. You should obtain visualizations similar to:

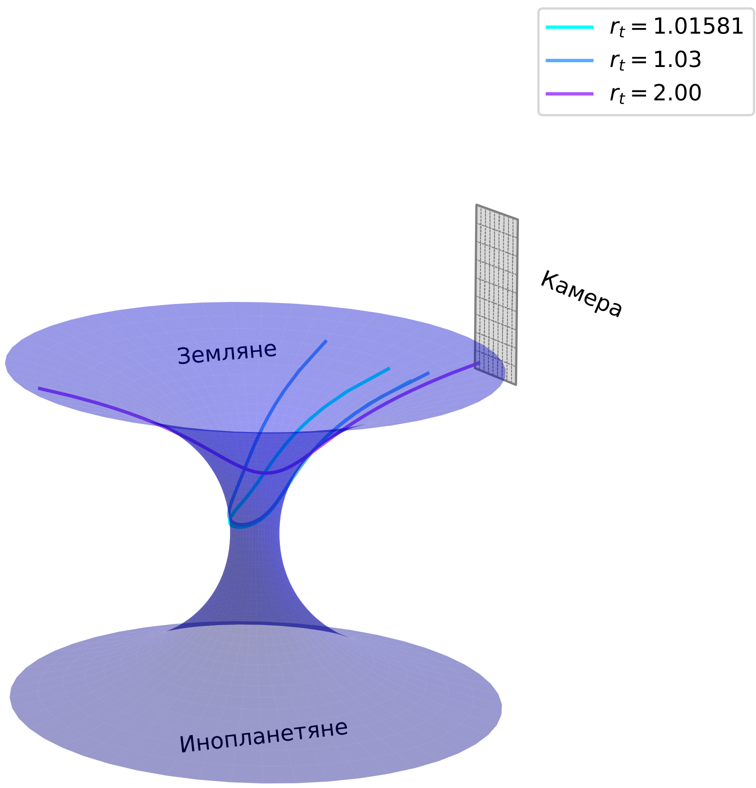



Figure 11.1: Regular geodesics near a wormhole for $b = 1$. The camera location is also shown.

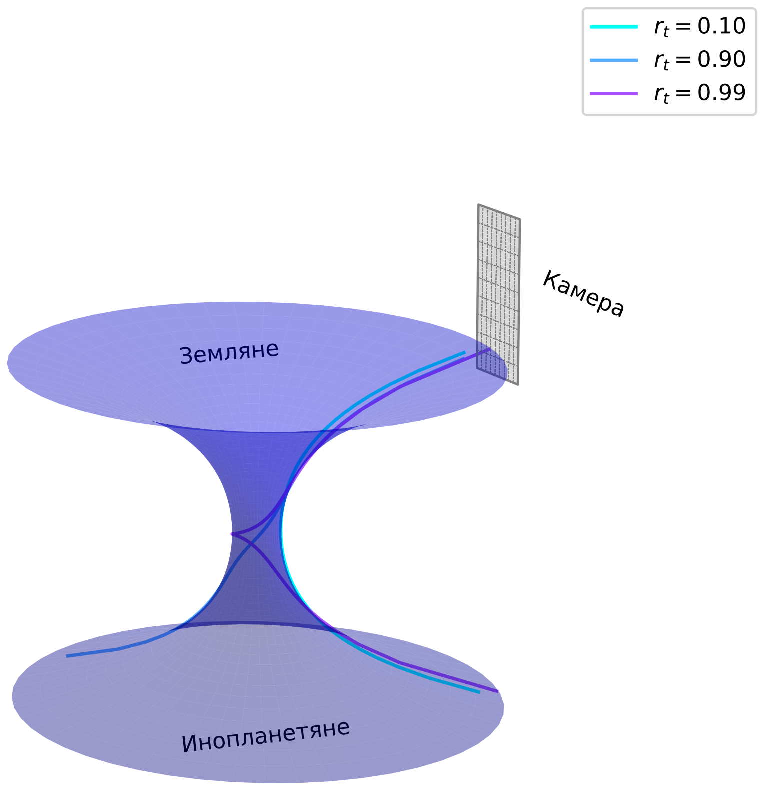



Figure 11.2: Singular geodesics near a wormhole for $b = 1$. The camera location is also shown.

8. Simulate or visualize what the camera sees near the wormhole. As the camera moves around the throat, how does the apparent position and brightness of stars change?

   *Optional: Create an animation showing the starry sky as seen through a wormhole. Use ray tracing, distorted projection, or existing software tools (e.g., Blender, Unity, Python with Matplotlib or Manim). Try showing how regular and singular geodesics bring light from different regions of the sky into the camera.*

9. Propose a conceptual experiment to detect a wormhole using imaging or

lensing data. What specific observational signatures (e.g., Einstein rings, duplicated or inverted stars, color shifts) could distinguish a wormhole from a black hole or standard gravitational lens?

## Deliverables

- Calculations and plots: Geodesic equations and solutions, including analytic expressions and plotted regular/singular trajectories near the wormhole.
- Visualization or animation: A sketch or animated sequence (optional but encouraged) showing how the sky would appear to a camera near a wormhole, including effects of geodesic bending.
- Scientific Report: A concise LaTeX-written report with derivations, trajectory analysis, visualization explanation, and a discussion of possible observational strategies for wormhole detection.
- Presentation: A 10-minute talk showcasing the physics of photon geodesics near wormholes, key visual effects, and proposed observational signatures.